Time series and error analysis with FOGMEx (`tsfit/tsview`), CATS and Hector

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Material from R. King, T. Herring, M. Floyd (MIT) and S. McClusky (now ANU)

Issues in GPS Error Analysis

- What are the sources of the errors ?
- How much of the error can we remove by better modeling ?
- Do we have enough information to infer the uncertainties from the data ?
- What mathematical tools can we use to represent the errors and uncertainties ?

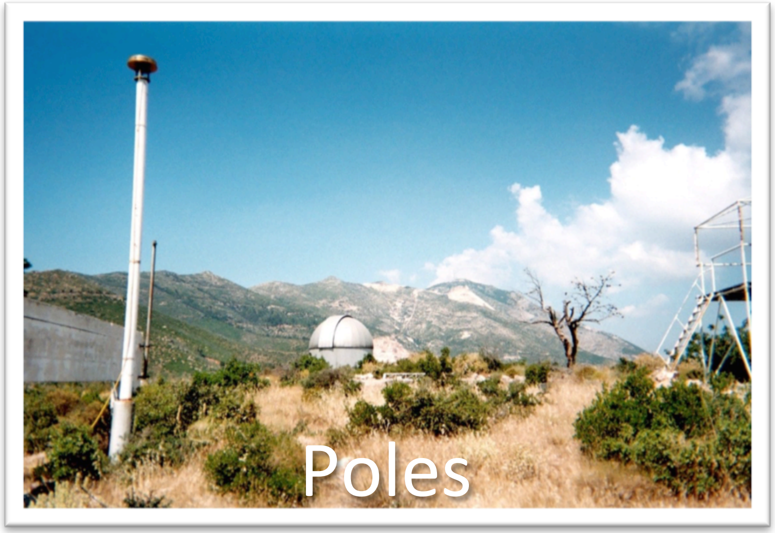
Determining the Uncertainties of GPS Parameter Estimates

- Rigorous estimate of uncertainties requires full knowledge of the error spectrum—both temporal and spatial correlations (never possible)
- Sufficient approximations are often available by examining time series (phase and/or position) and reweighting data
- Whatever the assumed error model and tools used to implement it, external validation is important

Sources of Error

- Signal propagation effects
  - Receiver noise
  - Ionospheric effects
  - Signal scattering (antenna phase center / multipath)
  - Atmospheric delay (mainly water vapor)
- Unmodeled motions of the station
  - Monument instability
  - Loading of the crust by atmosphere, oceans, and surface water
- Unmodeled motions of the satellites

Fixed antennas



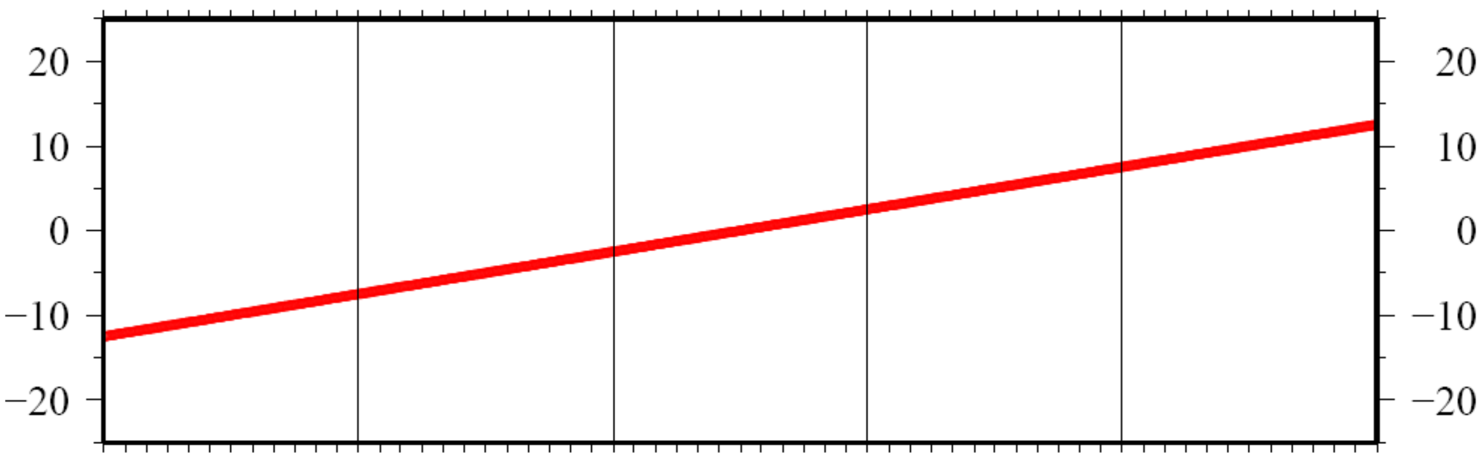
Time series characteristics

Time series components

observed position

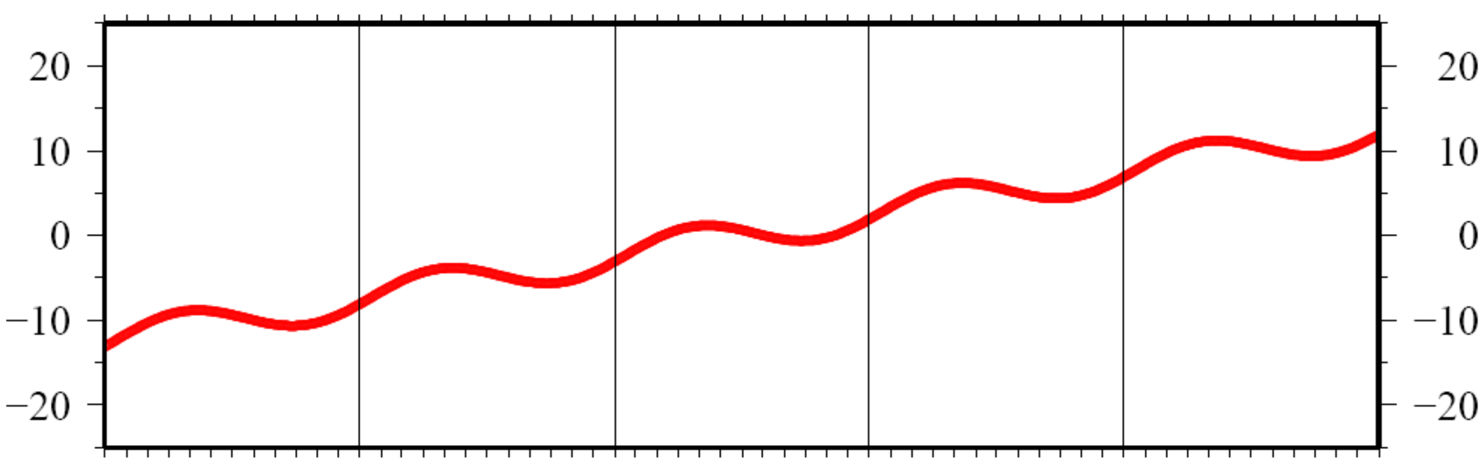
$$x^i = x_0^i + \overbrace{v^i(t - t_0)}^{\text{(linear) velocity term}}$$

initial position



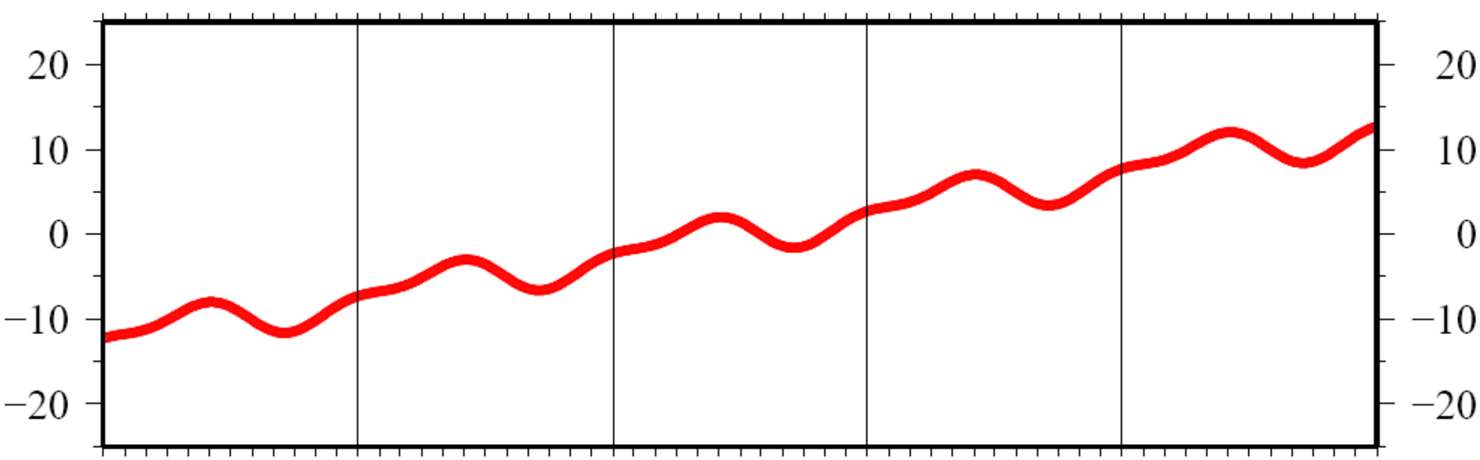
Time series components

$$\overset{\substack{\text{observed}\\\text{position}\\\downarrow}}{x^i} = \underset{\substack{\uparrow\\\text{initial}\\\text{position}}}{x_0^i} + \overbrace{v^i (t - t_0)}^{\substack{\text{(linear)}\\\text{velocity term}}} + \underbrace{A_0^i \cos\left(\frac{2\pi (t - t_0)}{T_0} - \tau_0\right)}_{\substack{\text{annual period}\\\text{sinusoid}}}$$



Time series components

$$\overset{\text{observed position}}{\overset{\downarrow}{x^i}} = \underset{\underset{\text{initial position}}{\uparrow}}{x_0^i} + \overbrace{v^i(t-t_0)}^{\text{(linear) velocity term}} + \underbrace{\underbrace{A_0^i\cos\left(\frac{2\pi(t-t_0)}{T_0}-\tau_0\right)}_{\text{annual period sinusoid}} + \underbrace{A_1^i\cos\left(\frac{2\pi(t-t_0)}{T_1}-\tau_1\right)}_{\text{semi-annual period sinusoid}}}_{\text{seasonal term}}$$

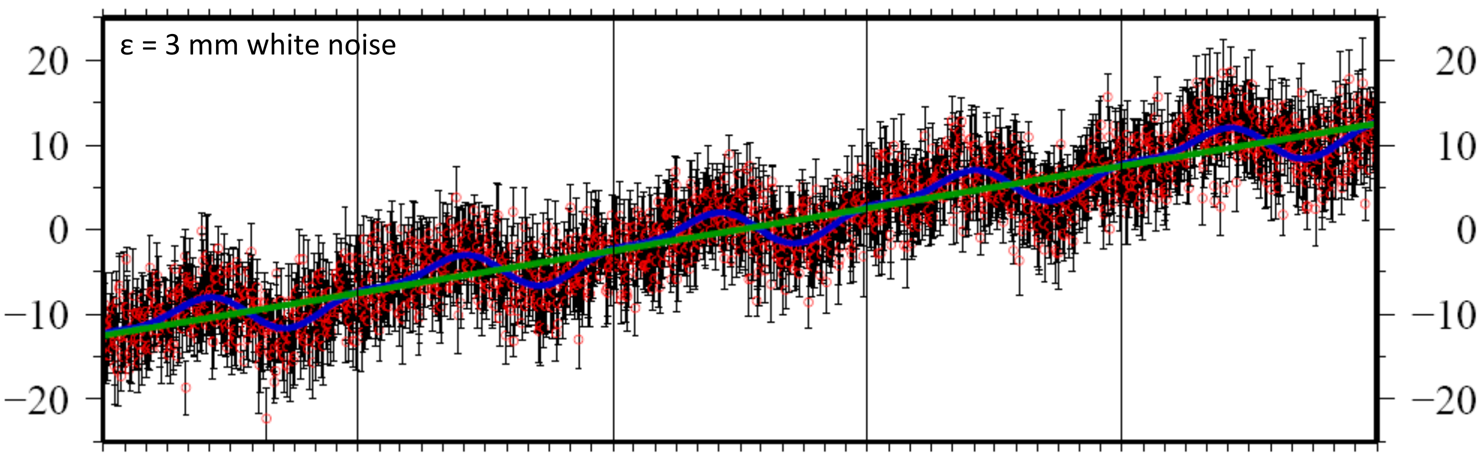


Time series components

observed position

initial position

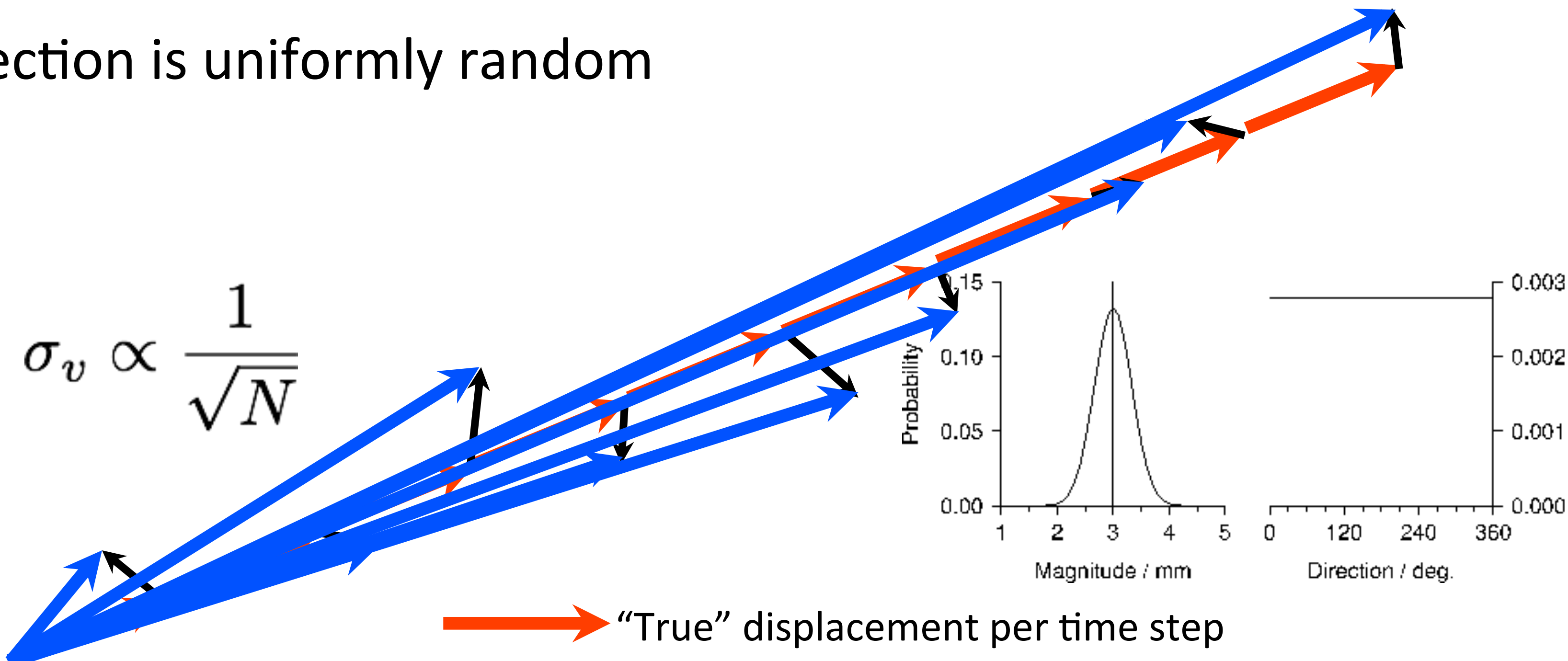
$$x^i = x_0^i + \overbrace{v^i (t - t_0)}^{\text{(linear) velocity term}} + \underbrace{\underbrace{A_0^i \cos\left(\frac{2\pi (t - t_0)}{T_0} - \tau_0\right)}_{\text{annual period sinusoid}} + \underbrace{A_1^i \cos\left(\frac{2\pi (t - t_0)}{T_1} - \tau_1\right)}_{\text{semi-annual period sinusoid}}}_{\text{seasonal term}} + \varepsilon$$



“White” noise

- Time-independent (uncorrelated)
- Magnitude has continuous probability function, e.g. Gaussian distribution
- Direction is uniformly random

$$\sigma_v \propto \frac{1}{\sqrt{N}}$$



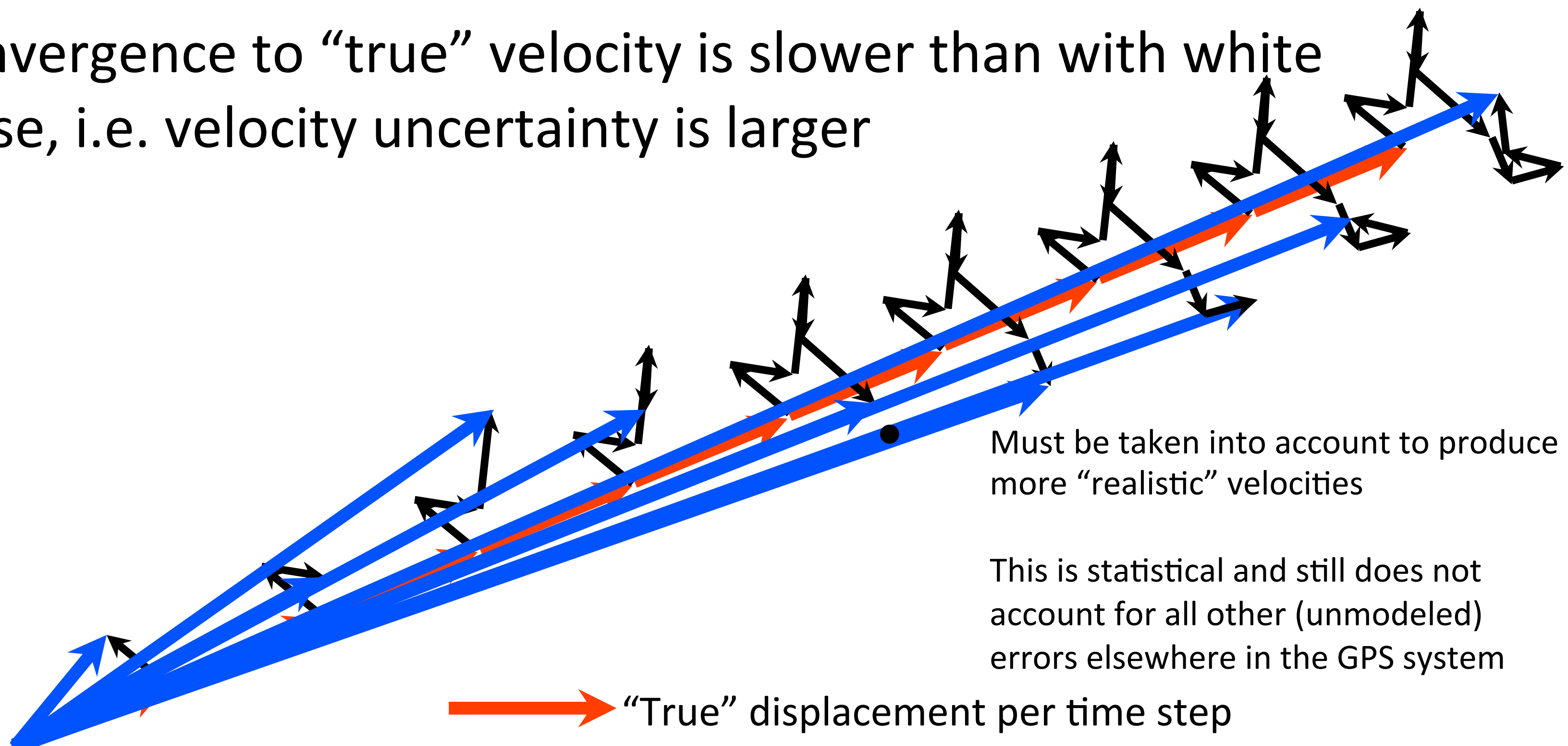
“True” displacement per time step

Independent (“white”) noise error

Observed displacement after time step t (v = d/t)

“Colored” noise

- Time-dependent (correlated): power-law, first-order Gauss-Markov, etc
- Convergence to “true” velocity is slower than with white noise, i.e. velocity uncertainty is larger



Must be taken into account to produce more “realistic” velocities

This is statistical and still does not account for all other (unmodeled) errors elsewhere in the GPS system

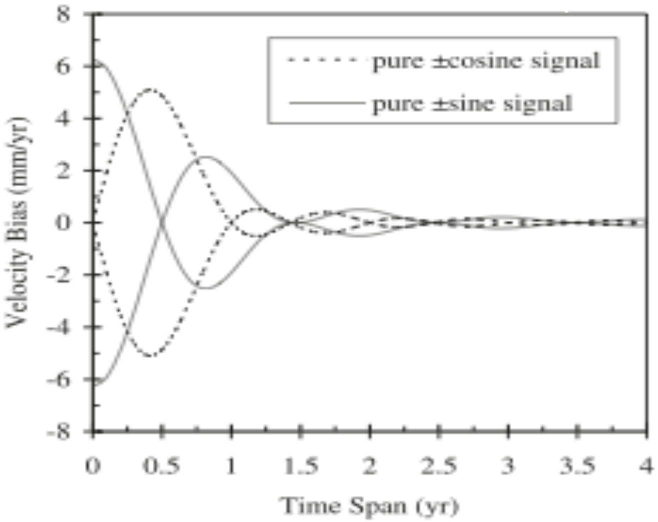
“True” displacement per time step

Correlated (“colored”) noise error*

Observed displacement after time step t (v = d/t)

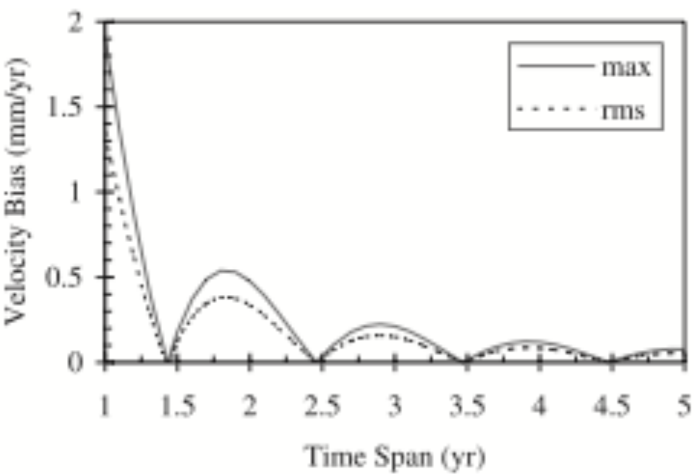
* example is “random walk” (time-integrated white noise)

Velocity Errors due to Seasonal Signals in Continuous Time Series



Theoretical analysis of a continuous time series by *Blewitt and Lavallee [2002]*

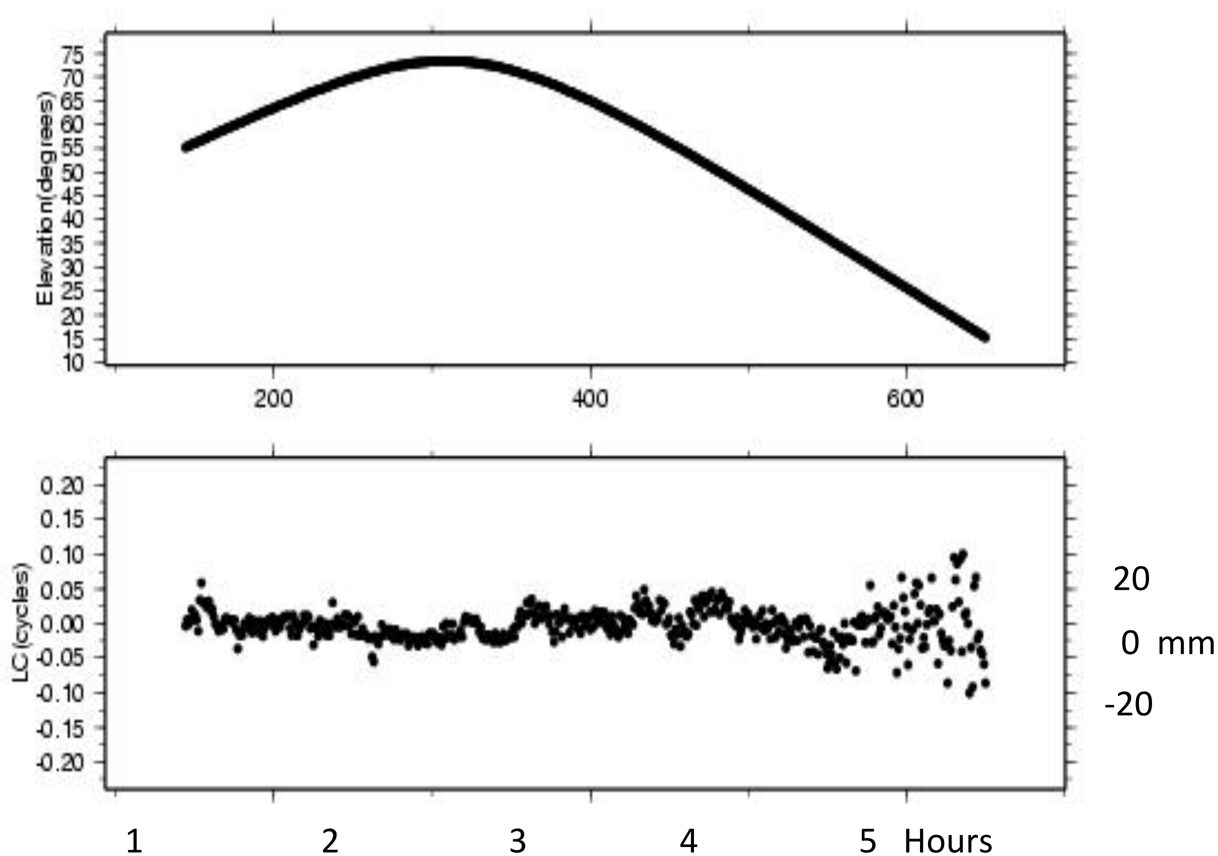
Top: Bias in velocity from a 1mm sinusoidal signal in-phase and with a 90-degree lag with respect to the start of the data span



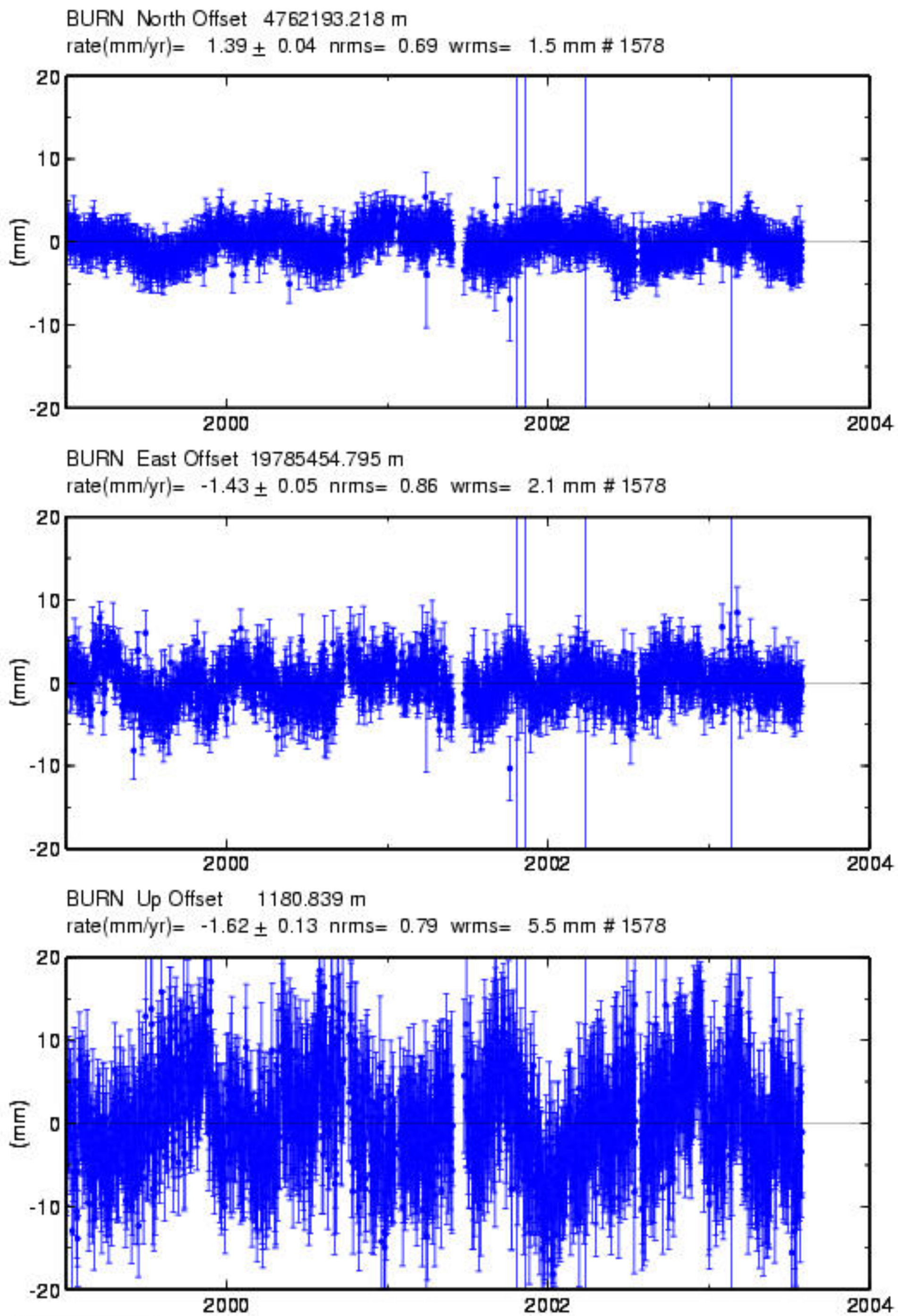
Bottom: Maximum and rms velocity bias over all phase angles

- The minimum bias is NOT obtained with continuous data spanning an even number of years
- The bias becomes small after 3.5 years of observation

Characterizing Phase Noise

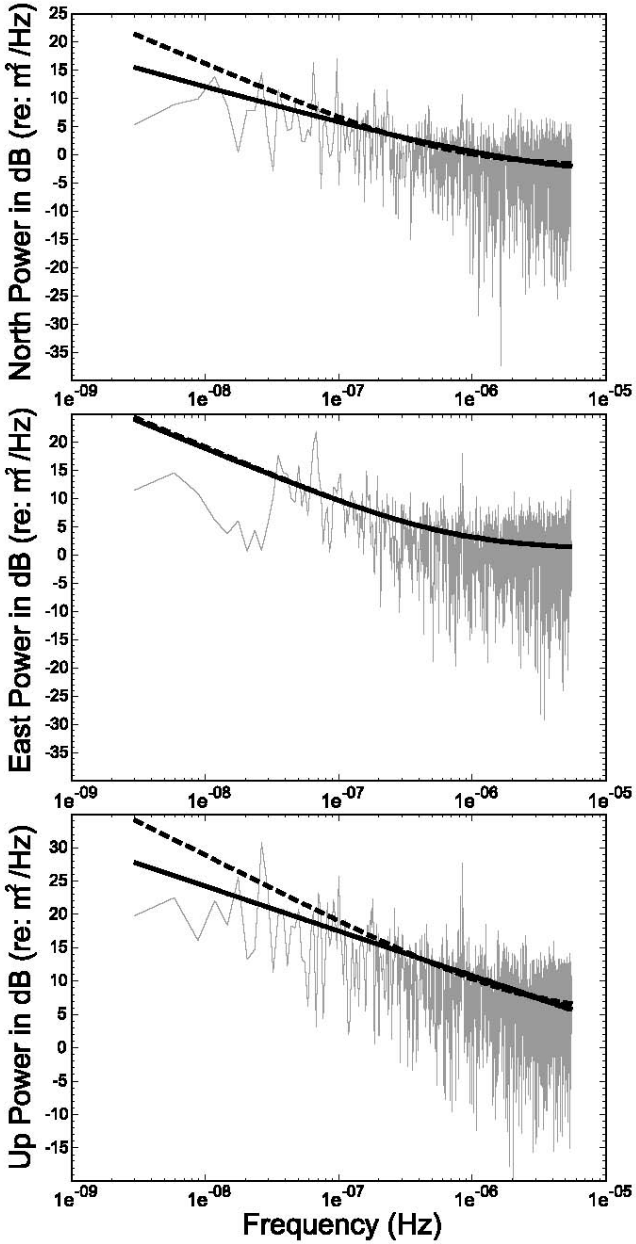


Elevation angle and phase residuals for single satellite



Characterizing the Noise in Daily Position Estimates

Note temporal correlations of 30-100 days and seasonal terms



Spectral Analysis of the Time Series
to Estimate an Error Model

Figure 5 from *Williams et al* [2004]: Power spectrum for common-mode error in the SOPAC regional SCIGN analysis. Lines are best-fit WN + FN models (solid=mean ampl; dashed=MLE)

Note lack of taper and misfit for periods > 1 yr

Summary of Spectral Analysis Approach

- Power law: slope of line fit to spectrum
  - 0 = white noise
  - -1 = flicker noise
  - -2 = random walk
- Non-integer spectral index (e.g. "fraction white noise" → 1 > k > -1)
- Good discussion in Williams [2003]
- Problems:
  - Computationally intensive
  - No model captures reliably the lowest-frequency part of the spectrum

CATS (Williams, 2008)

- Create and Analyze Time Series
- Maximum likelihood estimator for chosen model
  - Initial position and velocity
  - Seasonal cycles (sum of periodic terms) [optional]
  - Exponent of power law noise model
- Requires some linear algebra libraries (BLAS and LAPACK) to be installed on computer (common nowadays, but check!)

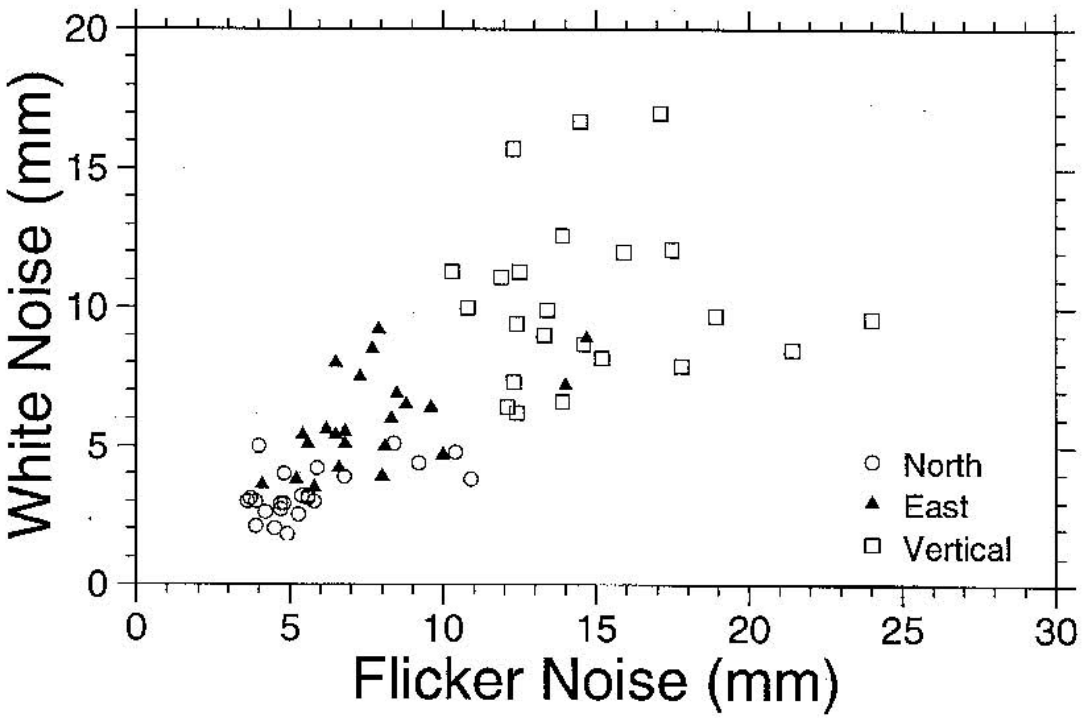
Hector (Bos et al., 2013)

- Much the same as CATS but faster algorithm
- Maximum likelihood estimator for chosen model
  - Initial position and velocity
  - Seasonal cycles (sum of periodic terms) [optional]
  - Exponent of power law noise model
  - Also
- Requires ATLAS linear algebra libraries to be installed on computer
- Linux package available but tricky to install from source due to ATLAS requirement

sh_cats/sh_hector

- Scripts to aid batch processing of time series with CATS or Hector
- Requires CATS and/or Hector to be pre-installed
- Outputs
  - Velocities in ".vel"-file format
  - Equivalent random walk magnitudes in "mar_neu" commands for sourcing in globk command file
- Can take a *long* time!

Short-cut (Mao et al, 1998):
Use white noise statistics (wrms) to predict the flicker noise



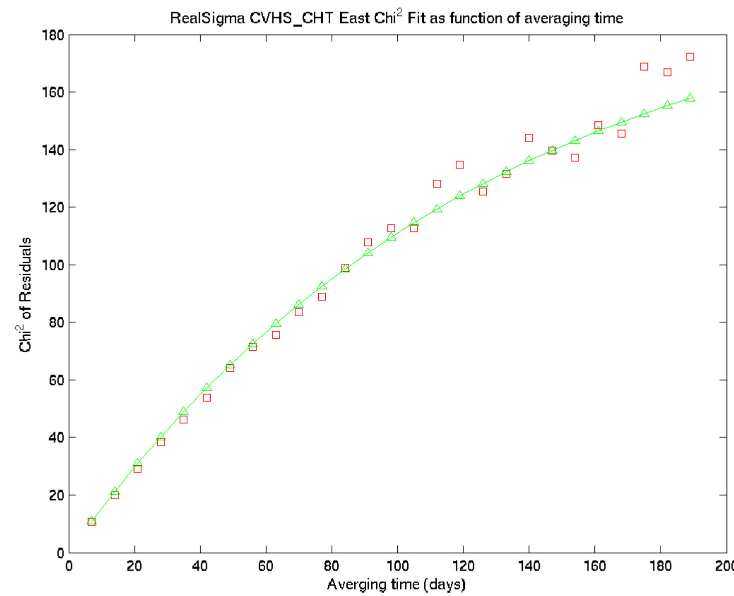
White noise vs flicker noise from *Mao et al.* [1999] spectral analysis of 23 global stations

“Realistic Sigma” Algorithm for Velocity Uncertainties

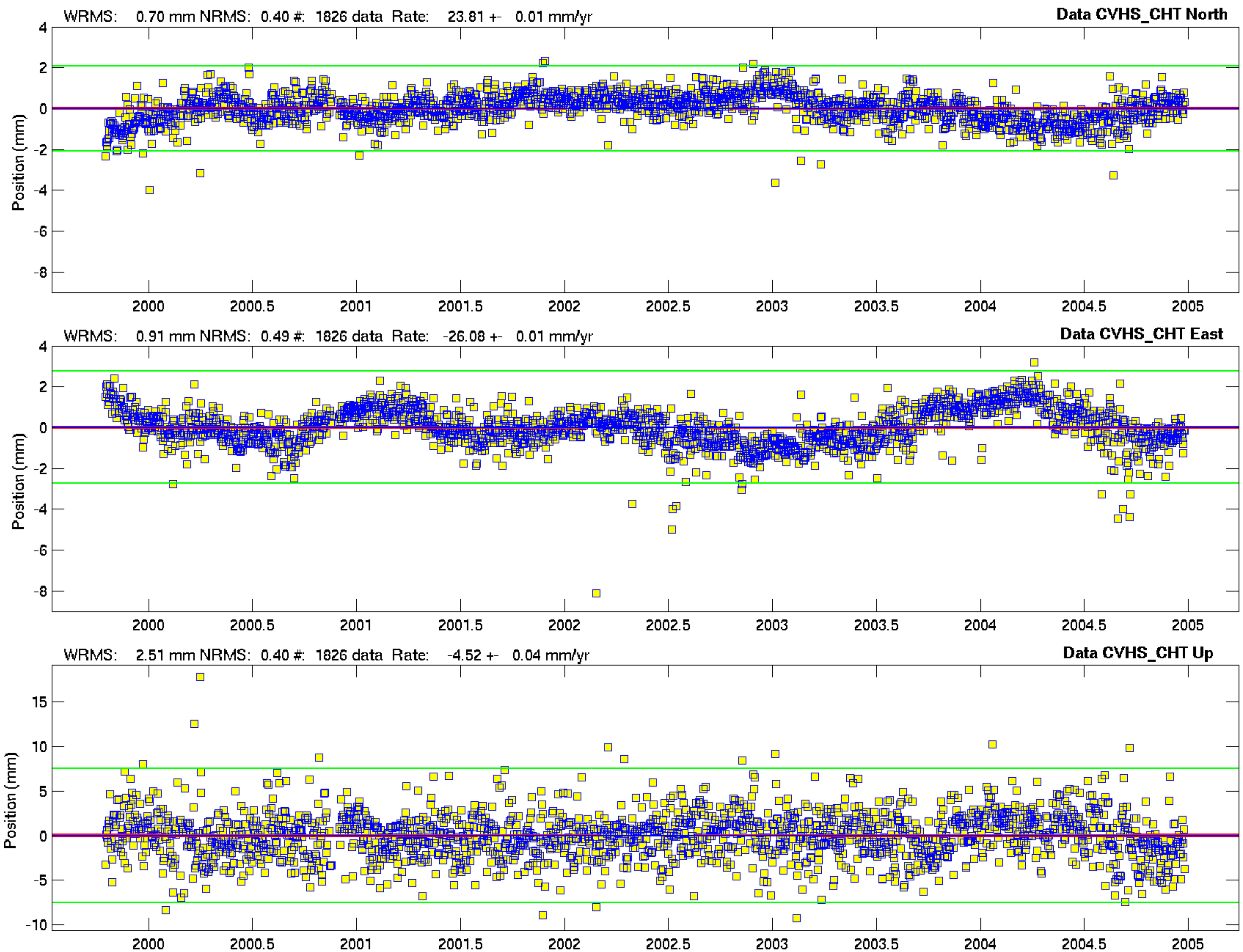
- Motivation: computational efficiency, handle time series with varying lengths and data gaps; obtain a model that can be used in *globk*
- Concept: The departure from a white-noise (sqrt n) reduction in noise with averaging provides a measure of correlated noise.
- Implementation:
  - Fit the values of chi2 vs averaging time to the exponential function expected for a first-order Gauss-Markov (FOGM) process (amplitude, correlation time)
  - Use the chi2 value for infinite averaging time predicted from this model to scale the white-noise sigma estimates from the original fit
  - and/or
  - Fit the values to a FOGM with infinite averaging time (i.e., random walk) and use these estimates as input to *globk* (mar_neu command)

Extrapolated variance (FOGMEx)

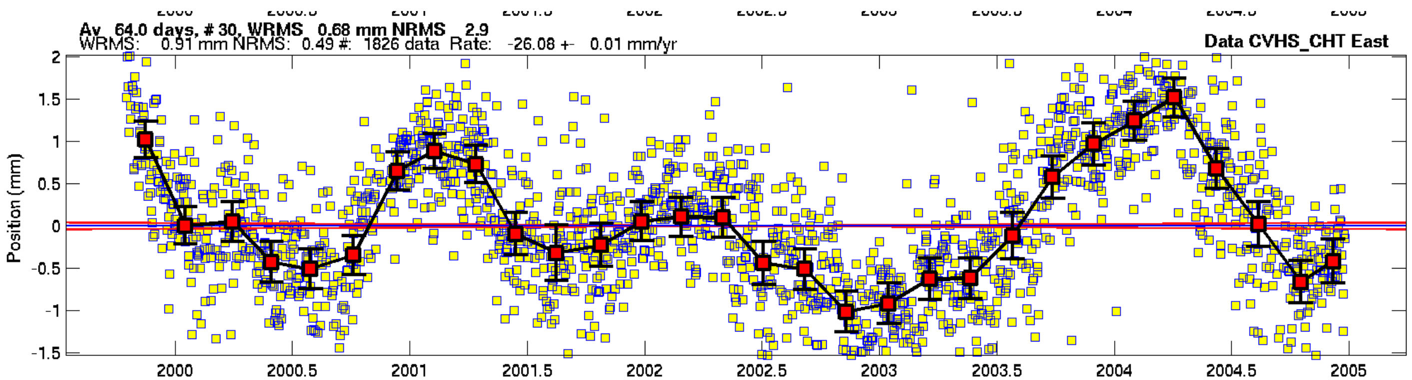
- For independent noise, variance $\propto 1/\sqrt{N_{data}}$
- For temporally correlated noise, variance (or χ^2/d.o.f.) of data increases with increasing window size
- Extrapolation to "infinite time" can be achieved by fitting an asymptotic function to RMS as a function of time window
  - χ^2/d.o.f. $\propto e^{-\sigma\tau}$
- Asymptotic value is good estimate of long-term variance factor
- Use "real_sigma" option in `tsfit`



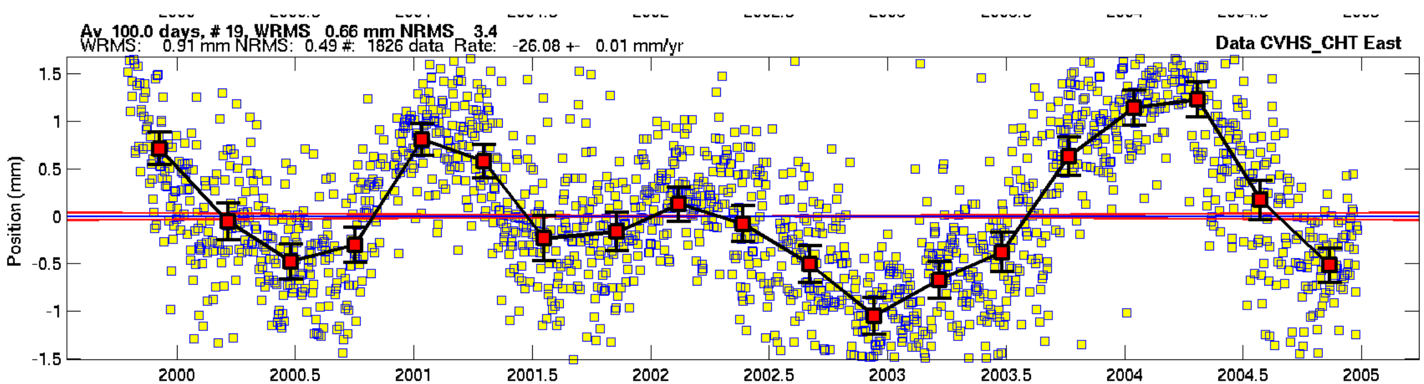
Understanding the RS algorithm: Effect of averaging on time-series noise



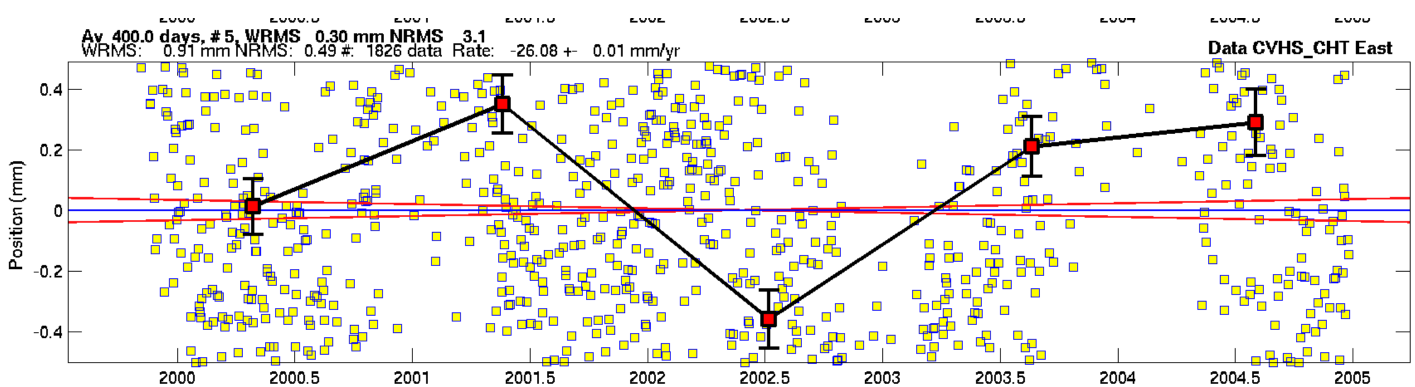
Same site, East component (daily wrms 0.9 mm nrms 0.5)



64-d avg
wrms 0.7 mm
nrms 2.0

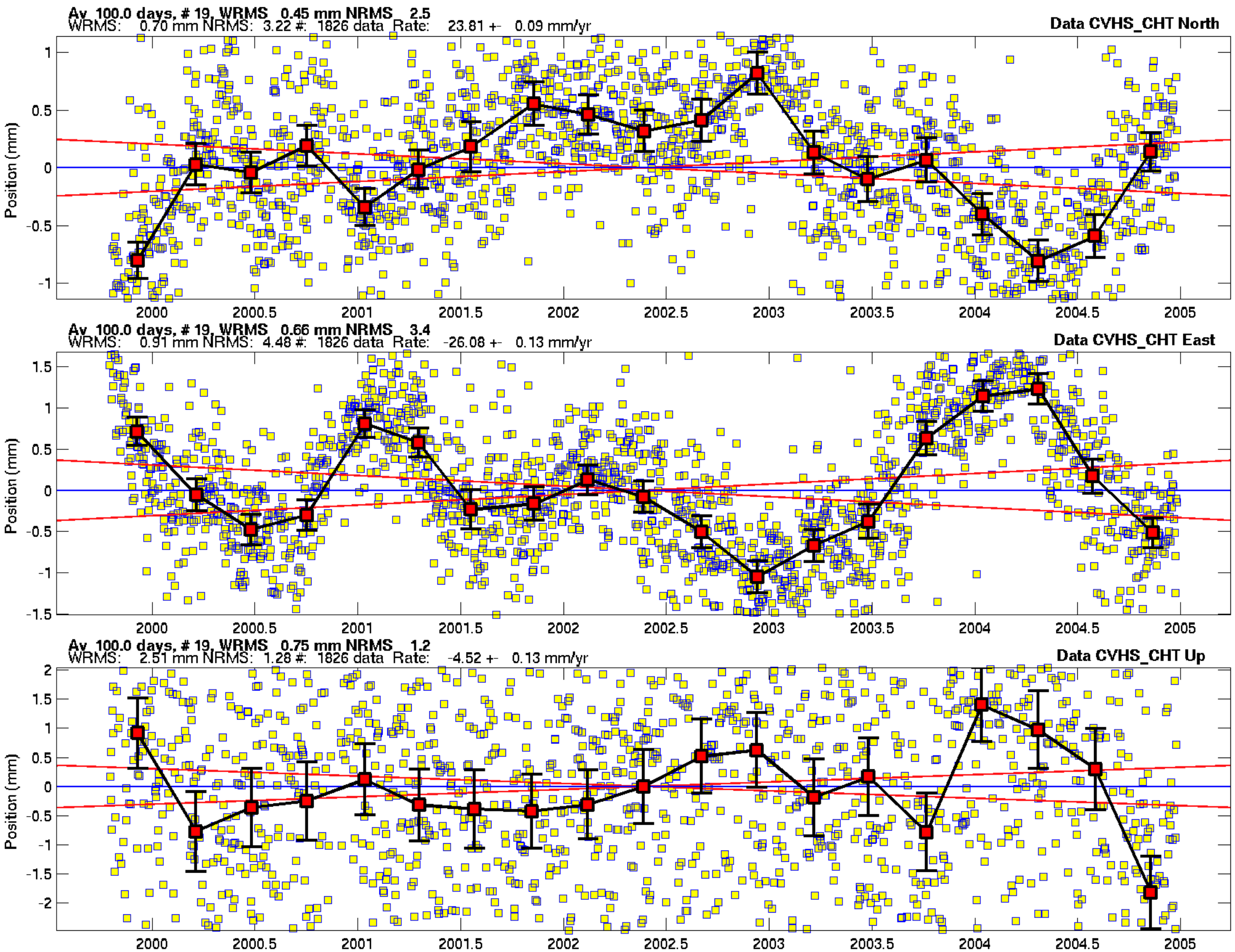


100-d avg
wrms 0.6 mm
nrms 3.4



400-d avg
wrms 0.3 mm
nrms 3.1

Using TSVIEW to compute and display the "realistic-sigma" results

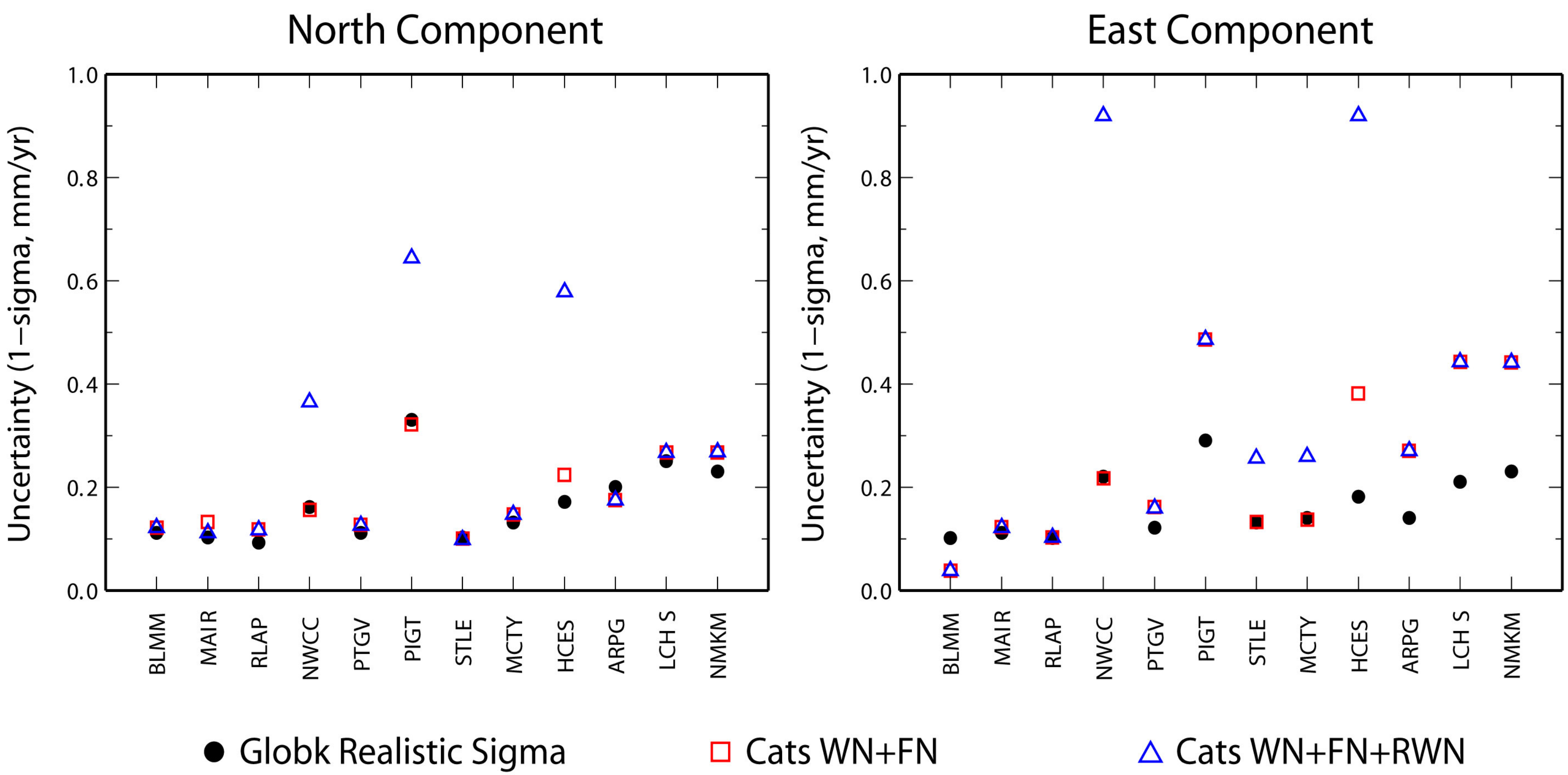


Note rate uncertainties with the "realistic-sigma" algorithm :

0.09 mm/yr N
0.13 mm/yr E
0.13 mm/yr U

Red lines show the 68% probability bounds of the velocity based on the results of applying the algorithm.

Comparison of estimated velocity uncertainties using spectral analysis (CATS) and Gauss-Markov fitting of averages (GLOBK)



Plot courtesy E. Calais

Summary of Practical Approaches

- White noise + flicker noise (+ random walk) to model the spectrum [Williams et al., 2004]
- White noise as a proxy for flicker noise [Mao et al., 1999]
- Random walk to model to model an exponential spectrum [Herring "realistic sigma" algorithm for velocities]
- "Eyeball" white noise + random walk for non-continuous data

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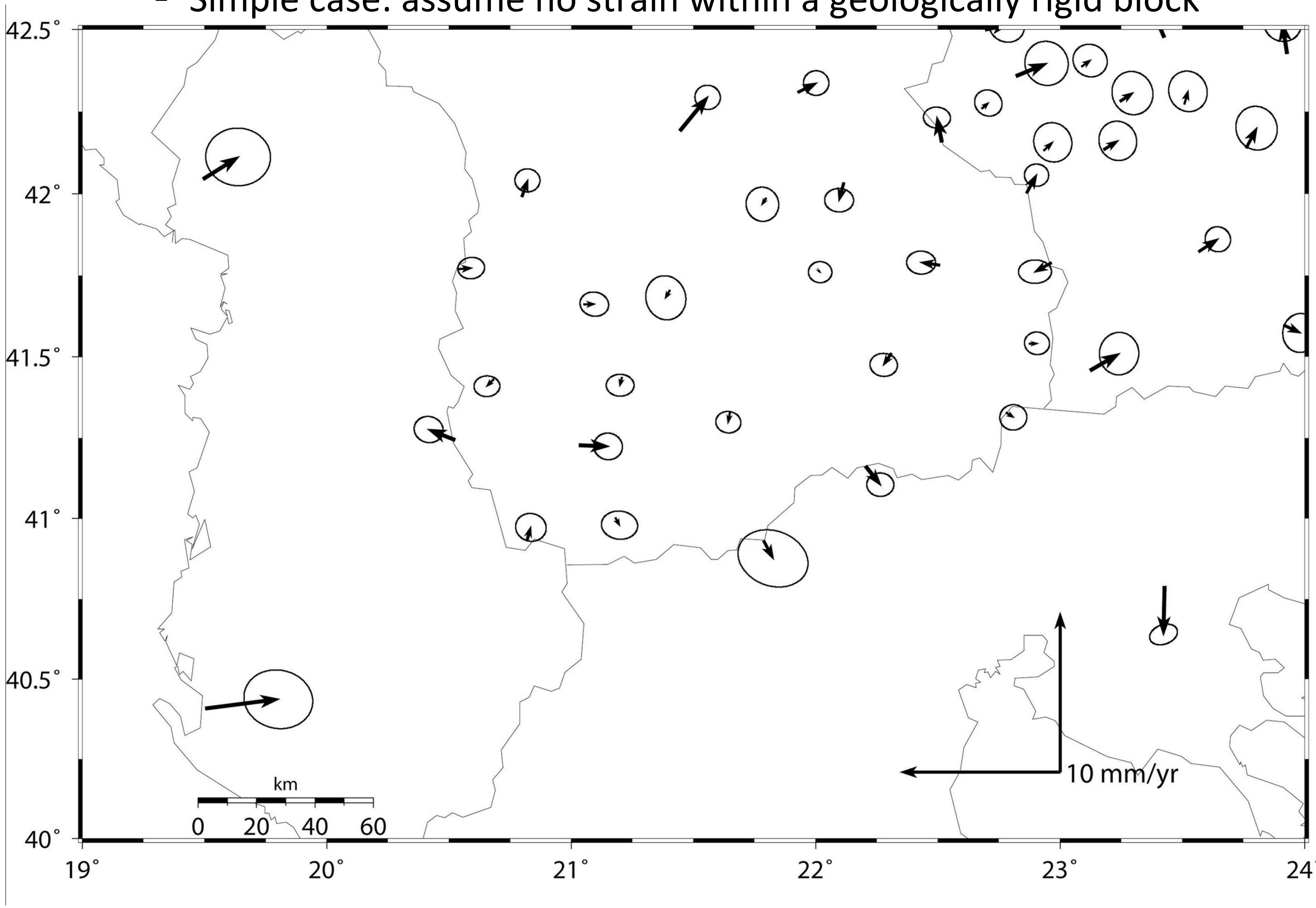
- Only the last two can be applied in GLOBK for velocity estimation
- All approaches require common sense and verification

External validation of velocity uncertainties by comparing with a model

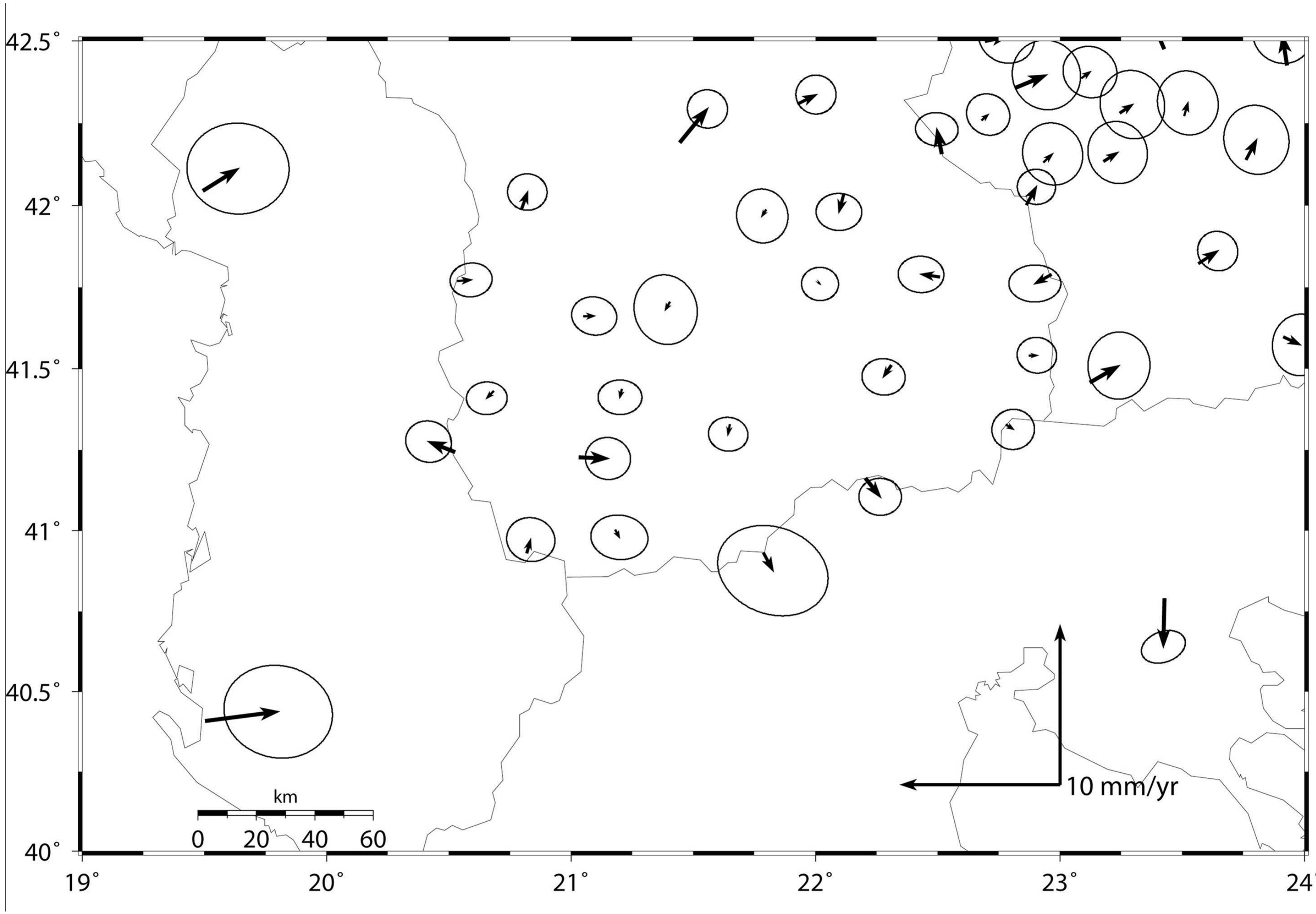
- Simple case: assume no strain within a geologically rigid block



GMT plot at 70% confidence

17 sites in central Macedonia: 4-5 velocities pierce error ellipses

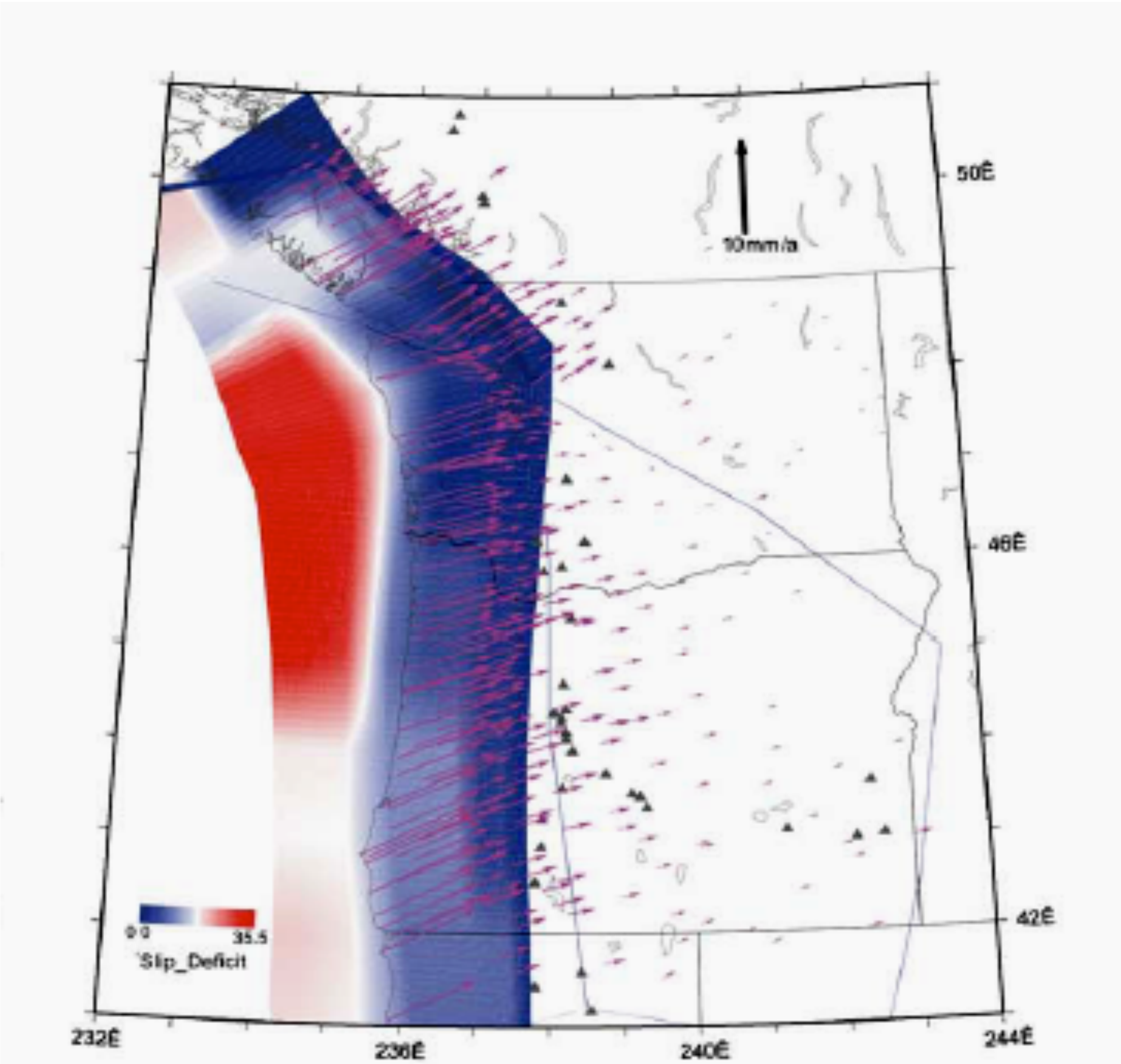
.. same solution plotted with 95% confidence ellipses



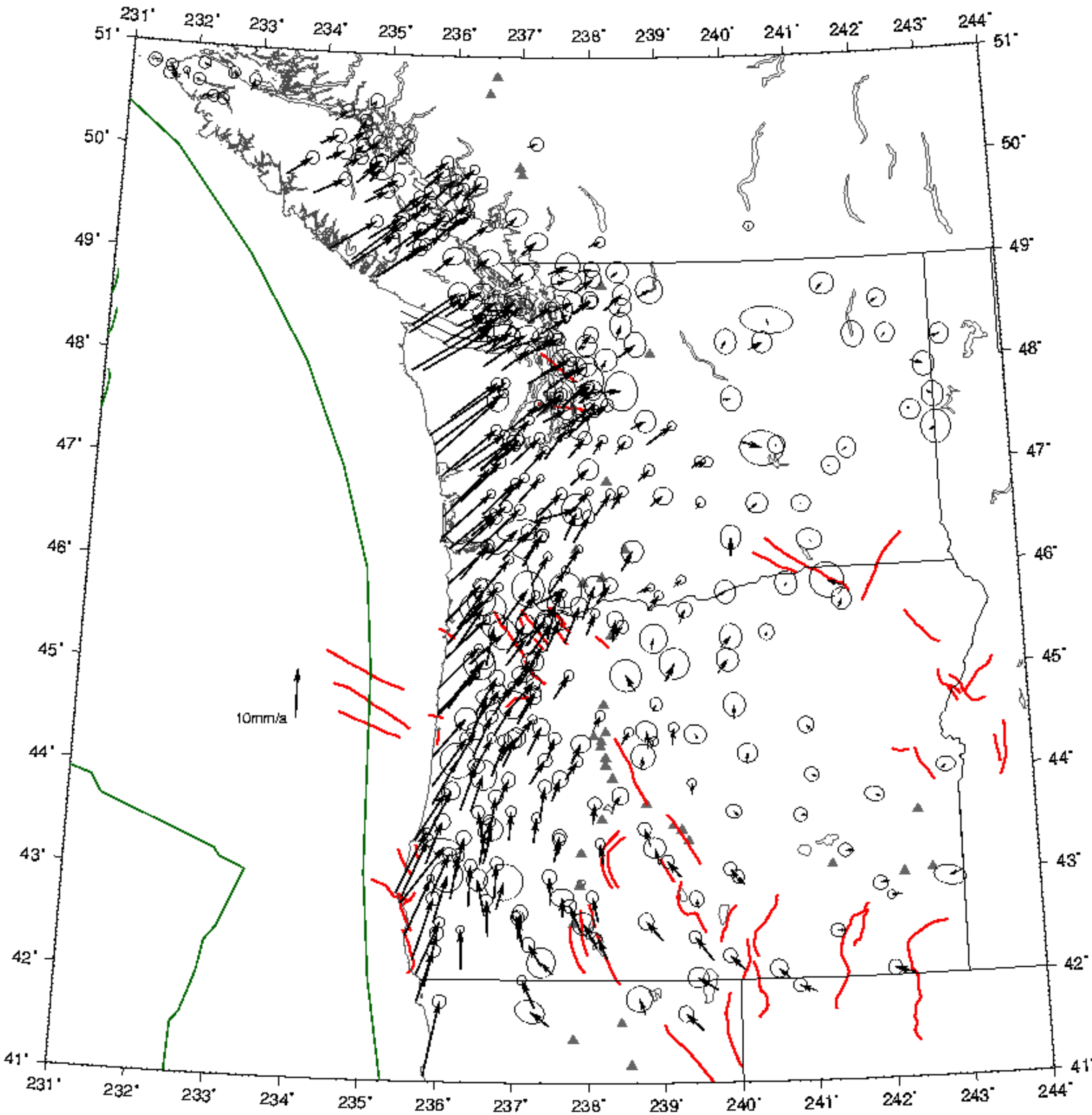
1-2 of 17 velocities pierce error ellipses

External validation of velocity uncertainties by comparing with a model

- a more complex case of a large network in the Cascadia subduction zone

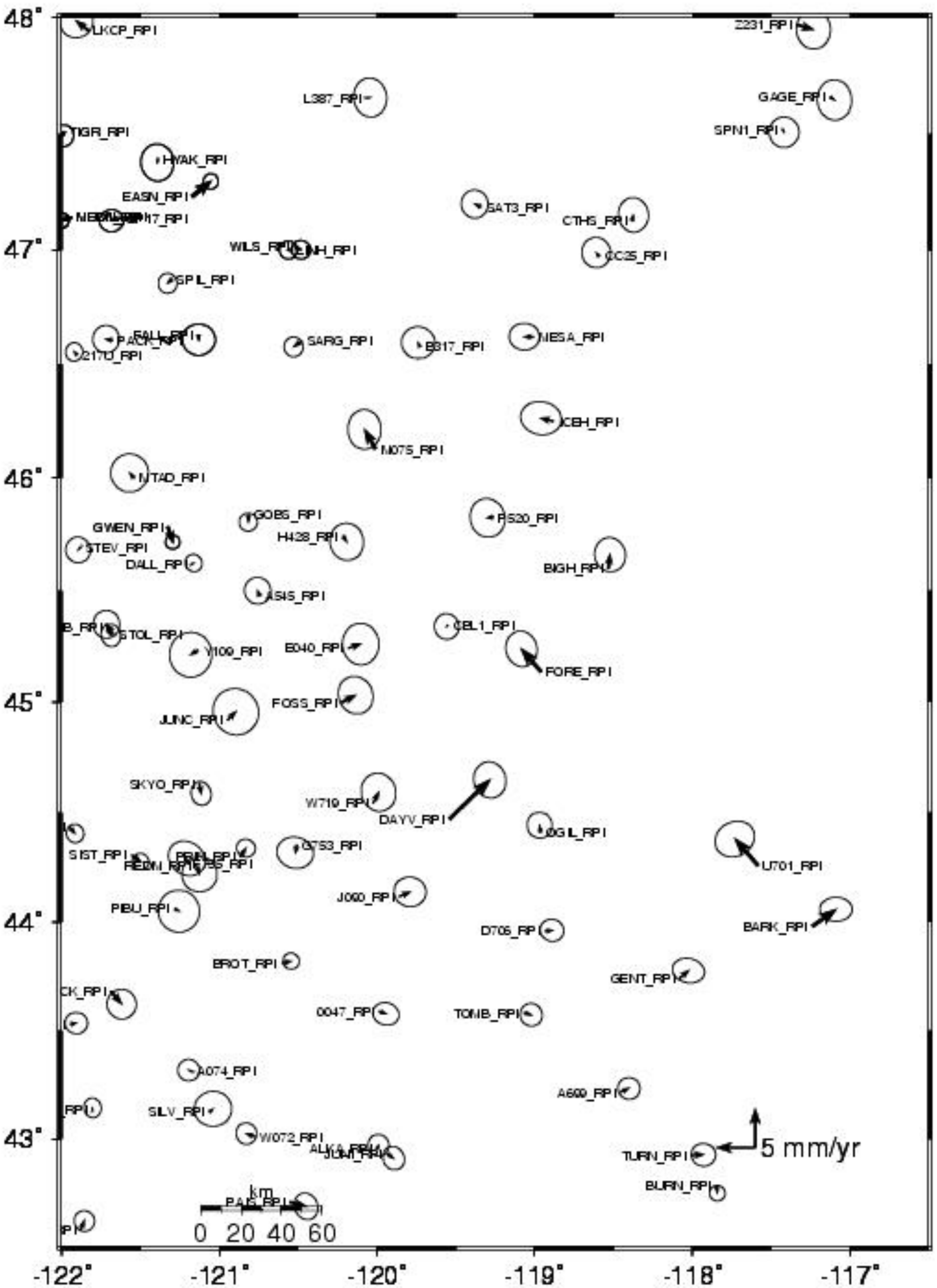


Colors show slipping and locked portions of the subducting slab where the surface velocities are highly sensitive to the model; area to the east is slowly deforming and insensitive to the details of the model



Velocities and 70% error ellipses for 300 sites observed by continuous and survey-mode GPS 1991-2004

Test area (next slide) is east of 238E

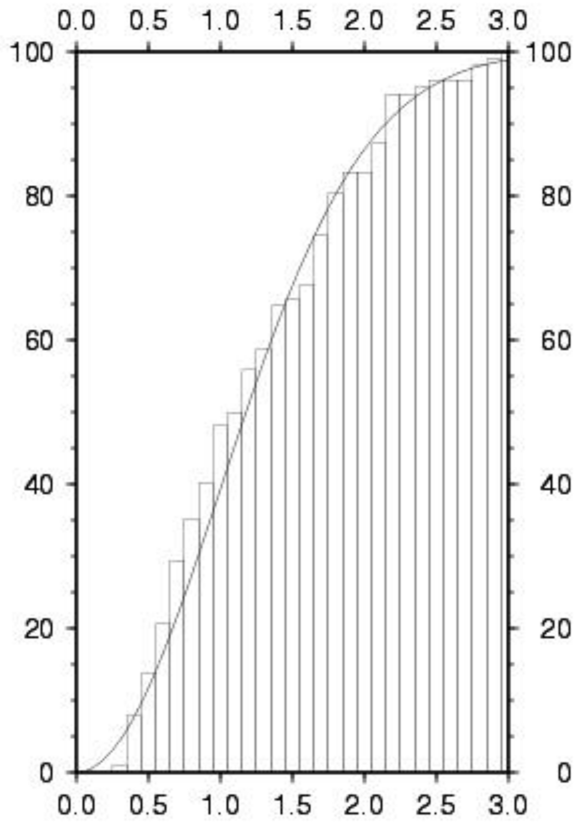


Residuals to elastic block model for 73 sites in slowly deforming region

Error ellipses are for 70% confidence:
13-17 velocities pierce their ellipse

Statistics of Velocity Residuals

Percent Within Ratio



Ratio (velocity magnitude/uncertainty)

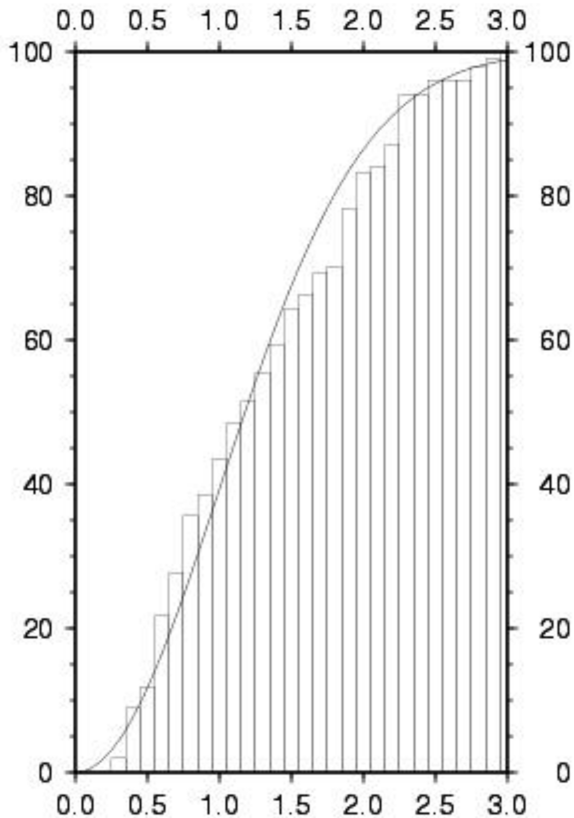
Cumulative histogram of normalized velocity residuals for Eastern Oregon & Washington (70 sites)

Noise added to position for each survey:
0.5 mm random
1.0 mm/sqrt(yr)) random walk

Solid line is theoretical for a chi distribution

Statistics of Velocity Residuals

Percent
Within
Ratio



Ratio (velocity magnitude/uncertainty)

Same as last slide but with a smaller random-walk noise added :

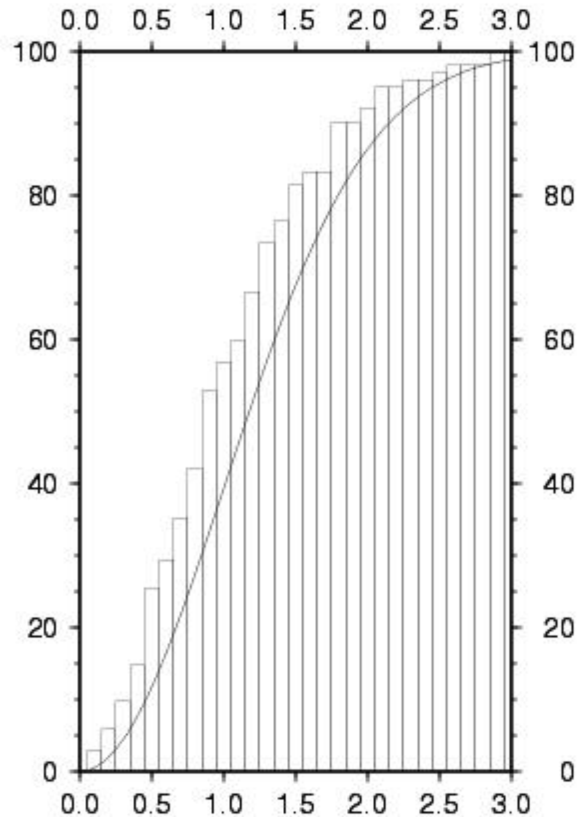
0.5 mm random
0.5 mm/yr random walk

(vs 1.0 mm/sqrt(yr)) RW for 'best' noise model)

Note greater number of residuals in range of 1.5-2.0 sigma

Statistics of Velocity Residuals

Percent
Within
Ratio



Ratio (velocity magnitude/uncertainty)

Same as last slide but with larger random and random-walk noise added :

2.0 mm white noise
1.5 mm/sqrt(yr)) random walk

(vs 0.5 mm WN and 1.0 mm/sqrt(yr)) RW for 'best' noise model)

Note smaller number of residuals in all ranges above 0.1-sigma

Summary

- All algorithms for computing estimates of standard deviations have various problems: Fundamentally, rate standard deviations are dependent on low frequency part of noise spectrum which is poorly determined.
- Assumptions of stationarity are often not valid
- "Realistic sigma" algorithm is a convenient and reliable approach to getting velocity uncertainties in *globk*
- Velocity residuals from a physical model, together with their uncertainties, can be used to validate the error model

Tools for Error Analysis in GAMIT/GLOBK

- GAMIT: AUTCLN reweight = Y (default) uses phase rms from postfit edit to reweight data with constant + elevation-dependent terms
- GLOBK
  - rename (eq_file) _XPS or _XCL to remove outliers
  - sig_neu adds white noise by station and span; best way to “rescale” the random noise component; a large value can also substitute for _XPS/_XCL renames for removing outliers
  - mar_neu adds random-walk noise: principal method for controlling velocity uncertainties
  - In the gdl files, can rescale variances of an entire h-file: useful when combining solutions from with different sampling rates or from different programs (Bernese, GIPSY)
- Utilities
  - tsview and tsfit can generate _XPS commands graphically or automatically
  - grw and vrw can generate sig_neu commands with a few key strokes
  - “Realistic sigma” algorithm implemented in tsview (MATLAB) and enfit/ensum; sh_gen_stats generates mar_neu commands for globk based on the noise estimates
  - sh_plotvel (GMT) allows setting of confidence level of error ellipses
  - sh_tshist and sh_velhist (GMT) can be used to generate histograms of time series and velocities.

References

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